MICROSTRIP ANALYSIS ON ANISOTROPIC AND/OR IHOMOGENEOUS SUBSTRATE WITH THE FINITE ELEMENT METHOD

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ABSTRACT

The finite element method has been applied to solve an open microstrip deposited on anisotropic and/or inhomogeneous substrate in the quasi static case. The results show a reduction in computing time and memory size when compared to other known methods.

INTRODUCTION

The finite element technique has been applied for solving the field distribution of an open microstrip deposited on anisotropic and/or inhomogeneous substrate in the quasi-static case. The domain under investigation has been divided into triangular elements. The element area has a logarithmic distribution as one goes away from the strip. A simplified net for the potential problem is shown in figure (1).

The potential function has been approximated by a linear interpolating polynomial on each element. Knowing the potential function , the electric field near the conductors can be determined, the electric field is integrated to find the charge on the strip or the ground plane with and without the substrate. Knowing these values, the transmission parameters can be determined[1].

To solve the potential problem one proceeds through the following steps :

- Nodes and elements are specified.
- The interpolating function for each element is defined.
- A system of linear equations is generated by minimizing a functional related to the potential problem.
- The system of equations is modified to fulfil the boundary conditions.
- The system of equations is solved to get the nodal potentials which define the potential function.

MATHEMATICAL FORMULATION

The potential function φ satisfies the general form of Laplace equation give by :

$$\nabla \cdot (\tilde{\varepsilon}\, \nabla \varphi) = 0 \qquad (1)$$

where $\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{pmatrix}$

from the calculus of variations one can have the functional:

$$\chi = \int_V F(x,y,z,\varphi,\varphi_x,\varphi_y,\varphi_z)\, dV$$

where φ_x , φ_y , φ_z are the first derivatives of the potential function w.r.t. the coordinates x,y and z respectively.

F is to be determined such that small arbitraty change in F will not change the value of χ . Taking small change in χ

$$\delta\chi = \int_V \Big[\frac{\partial F}{\partial \varphi}\delta\varphi + \frac{\partial F}{\partial \varphi_x}\frac{\partial}{\partial \varphi_x}(\delta\varphi) + \frac{\partial F}{\partial \varphi_y}\frac{\partial}{\partial y}(\partial\varphi) + \frac{\partial F}{\partial \varphi_z}\frac{\partial}{\partial z}(\delta\varphi)\Big]\, dV \qquad (2)$$

It can be easily proved that

$$\delta\chi = \int_V \Big[\frac{\partial F}{\partial \varphi} - \frac{\partial}{\partial x}\Big(\frac{\partial F}{\partial \varphi_x}\Big) - \frac{\partial}{\partial y}\Big(\frac{\partial F}{\partial \varphi_y}\Big) - \frac{\partial}{\partial z}\Big(\frac{\partial F}{\partial \varphi_z}\Big)\Big]\delta\varphi\, dV + \int_V \Big[\frac{\partial}{\partial x}\Big(\frac{\partial F}{\partial \varphi_x}\delta\varphi\Big) + \frac{\partial}{\partial y}\Big(\frac{\partial F}{\partial \varphi_y}\delta\varphi\Big) + \frac{\partial}{\partial z}\Big(\frac{\partial F}{\partial \varphi_z}\delta\varphi\Big)\Big] dV$$

Applying Grauss theorem on the second term of $\delta\chi$:

$$\delta\chi = \int_V \Big[\frac{\partial F}{\partial \varphi} - \frac{\partial}{\partial x}\Big(\frac{\partial F}{\partial \varphi_x}\Big) - \frac{\partial}{\partial y}\Big(\frac{\partial F}{\partial \varphi_y}\Big) - \frac{\partial}{\partial z}\Big(\frac{\partial F}{\partial \varphi_z}\Big)\Big]\delta\varphi\, dV + \oint_A \Big(l_x \frac{\partial F}{\partial \varphi_x} + l_y \frac{\partial F}{\partial \varphi_y} + l_z \frac{\partial F}{\partial \varphi_z}\Big]\delta\varphi\, dA \qquad (3)$$

Where A is the closed surface surrounding the volume V and l_x, l_y, l_z are direction cosines of the normal to the surface of the elements.

$\delta\chi$ will be zero if the bracketed terms in eq. (3) equal to zero.

Choosing

$$F = \tfrac{1}{2}\varepsilon_{xx}\varphi_x^2 + \varepsilon_{xy}\varphi_x\varphi_y + \tfrac{1}{2}\varepsilon_{yy}\varphi_y^2 \qquad (4)$$

and substituting F in the first backeted term of $\delta\chi$ leads to Laplace equation given by eq. (1) while the second term gives the boundary condition at the interface between two medii ($D_{1n} = D_{2n}$) where D is the electric displacement vector.

It is clear that the minimization of the functional χ given by

$$\chi = \frac{1}{2}\int_V \Big[\varepsilon_{xx}\varphi_x^2 + 2\varepsilon_{xy}\varphi_x\varphi_y + \varepsilon_{yy}\varphi_y^2\Big] dV \qquad (5)$$

necessitates that φ must satisfy Laplace equation given by eq. (1) and the boundary condition at the interface between two different medii.

COMPUTER IMPLEMENTATION

Minimizing the functional χ results in a system of linear homogeneous equations[1] given by

$$[K][\varphi] = 0 \qquad (6)$$

where [K] is a symmetric and sparce mutrix, its elements depends on the nodal coordinates and $[\Phi]$ is the nodal potentials. The system of equations (6) is modified to fulfil the boundary condition of both the strip conductor and the ground plane.

Resulting in

$$[K][\Phi] = [F] \qquad (7)$$

In the matrix [K] non of the diagonal terms equals zero. It is a sparce matrix which has a limited bandwidth. The bandwidth equals the largest difference between the node numbers in a single element plus one.

The upper triangular terms are stored in a one dimensional matrix [BK] . This is done in the following way :

$$\overset{\longleftarrow\; b \;\longrightarrow}{\begin{bmatrix} K_{11} & K_{12} & K_{13} & 0 & 0 & 0 \\ 0 & K_{22} & K_{23} & K_{24} & 0 & 0 \\ 0 & 0 & K_{33} & K_{34} & K_{35} & 0 \\ 0 & 0 & & K_{44} & K_{ij} & \cdots 0 \\ 0 & 0 & \cdots & \cdot & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdots & K_{pp} \end{bmatrix}} \Rightarrow \begin{bmatrix} BK_1 = K_{11} \\ BK_2 = K_{12} \\ BK_3 = K_{22} \\ \cdots = \cdots \\ BK_m = K_{ij} \\ BK_l = K_{pp} \end{bmatrix} \qquad (8)$$

The relation between the indices (i , j) and the index (m) of the one dimensional matrix is given by[1] :

$$m = (i-1)(b-1) + j \quad , \; 1 \leqslant i \leqslant p-b+1$$

$$m = (i-1)(2p-i)/2 - w \quad , \; p-b+1 < i \leqslant p \qquad (9)$$

where $w = (p-b)(p-b+1)/2$

p : total number of nodes

b : bandwidth of the matrix [K]

using formulas (9), the core-size for the matrix is reduced from 320 K-word to 18 k-word for 400 nodal points.

The bandwidth of the matrix [K] can be reduced by proper numbering of nodes. This can be done as follows :

1. The numbering starts scanning the shortest dimension.

or

2. Starting from any arbitrary corner and proceeding through the adjacent nodes.

Solving the system of equations(7), one can get the nodal potentials and then all other line parameters.

APPLICATIONS OF THE METHOD

1. Microstrip deposited on a sapphire substrate (anisotropic medium)

   The method has been applied to solve this case having $\varepsilon_{xx} = 9.4$, $\varepsilon_{yy} = 11.6$, $\varepsilon_{yx} = \varepsilon_{yx} = 0$, results obtained gave good agreement Fig. 2 with the results given by Owens[3], who applied the finite difference method, and his method is applicable only if optical axis are perpendicular to ground plane, i.e. $\varepsilon_{xx} = \varepsilon_{xy} = 0$. The finite element method has the following advantages in comparison with the finite difference method :

   a) The case $\varepsilon_{xy} = \varepsilon_{yx} \neq 0$ can be solved

   b) Reduction in computing time and core size

2. Microstrip deposited on a groove of high dielectric material : Fig.3

   The method has been applied and results are shown in Fig. 3, this configuration is of great importance in Microwave integrated circuits and cannot be solved by conformal mapping method.

3. Microstrip on multilayer substrates(Inhomogeneous) :

   The method can be easily applied to solve the case of microstrip on a multilayer dielectric as well as on an inhomogeneous medium

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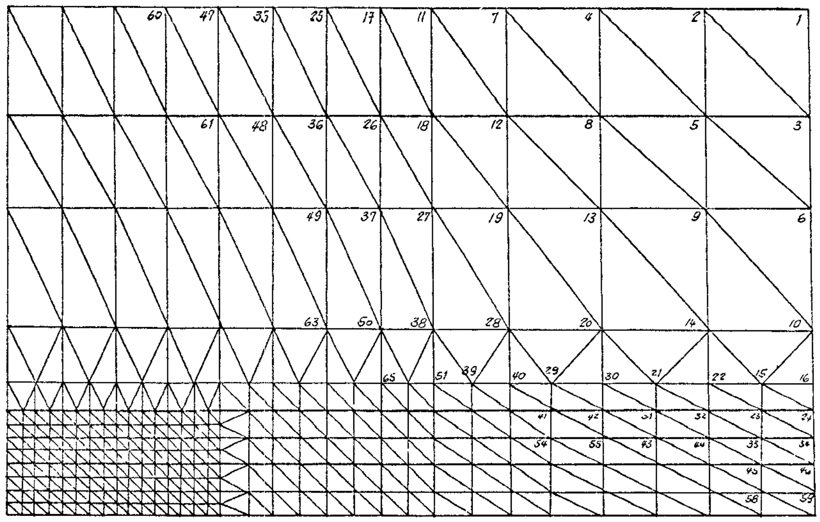

Fig. 1
Simplified net for nodes and elements (half of the domain is drawn)

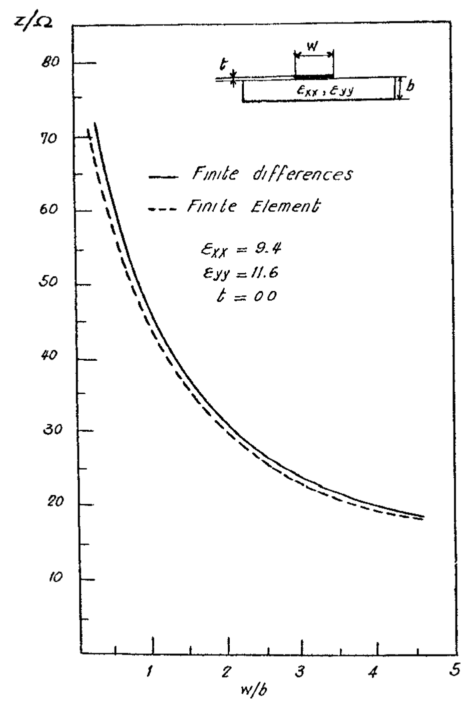

Fig 2
Impedance versus w/b for microstrip on a sapphire substrate.

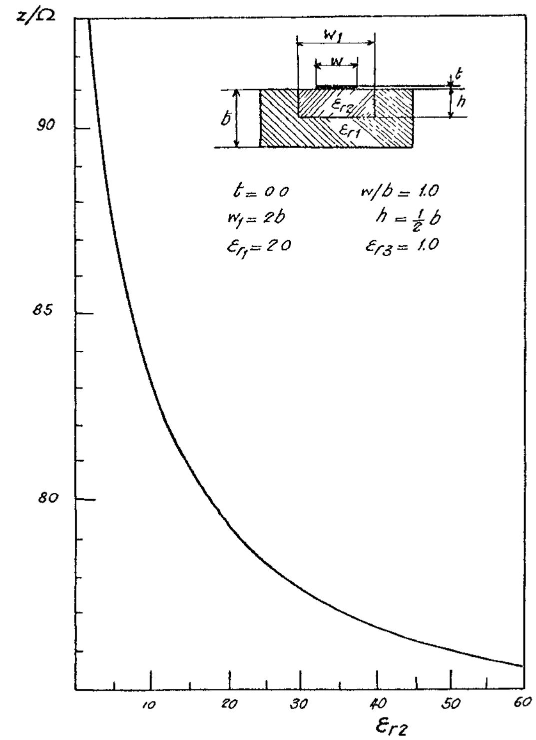

Fig. 3
The characteristic impedance of microstrip deposited on a Groove of high dielectric material.